Meadows or Malls?

(Using Matrices to Solve Systems)

Objective

Students explore the use of matrices and graphing calculators to solve a complex linear programming problem.

Overview of the Lesson

Students use matrices and technology to solve the *Meadows or Malls?* problem, a linear programming problem with six variables. The students begin by reviewing homework problems in which they relate the solvability of a system of linear equations to the invertibility of the coefficient matrix. The students use graphing calculators to solve linear systems. The final activity of the lesson involves reviewing all of the equations and constraints of the *Meadows or Malls?* problem, refining those considerations, using the graphing calculator to solve the linear systems involving six equations and six variables, and then testing those solutions in the cost equation to determine the best division of land use for the city.

Materials

- graphing calculators and overhead unit
- overhead projector
- *Meadows or Malls?* activity sheets
- *Inverses and Equations* activity sheets
- *TI's Again* activity sheets
- *Meadows or Malls?: Now We Are Ready* activity sheets

Procedure

1. **Small Group Discussion about Homework on *Inverses and Equations***: Have students work in small groups to review their homework activity, *Inverses and Equations*. While they are sharing their solutions, help them notice that different approaches have brought about the same solution.

2. **Class Discussion about Homework on *Inverses and Equations***: Have several students present answers to various problems. An important fact to bring out in this discussion is that some of the problems have unique solutions, but others do not. A system having a unique solution is consistent and independent. A system having an infinite number of solutions is consistent and dependent. A system having no solution is described as inconsistent.

   Have students share ideas about which matrices have inverses. Help them realize that a matrix in which one row is a multiple of another does not have an inverse and is related to either dependent or inconsistent systems of equations. If the matrix does have an inverse, then there is a unique solution to the linear system.

3. **Small Groups Use the Graphing Calculator to Solve Linear Systems**: In this activity, students use the matrix capability of the TI-83. First they define the coefficient matrix, and then they multiply its inverse by the constant matrix to find the solution to the linear systems. Hand out the *TI's Again* activity sheet. Have students work in groups to first read the activity sheet, and then complete the problems, discussing their solutions.

4. **Class Discussion about the *TI's Again* Activity**: During the class discussion of the problems, bring the students back to the issue of what it means for a system to have no unique solution. Relate this to the error message that the students may have encountered on the calculator.

   Another point to bring out in the discussion is the meaning of not having a solution to a particular system as that relates to solving a linear programming problem. The points of interest in a linear programming problem are the corners of the feasible region. A system that does not have a unique solution will not produce a corner point and therefore is not important to the solution of the linear programming problem.

5. **Small Group Discussion about the *Homework: Meadows or Malls? Now We Are Ready!*** Several days earlier the students had listed the constraints of the unit problem. Each group had a list of twenty-eight combinations to start from. This came from the fact that there are six constraints from the situation.

(1) $G_R + G_D = 300$
(2) $A_R + A_D = 100$
(3) $M_R + M_D = 150$
(4) $G_D + A_D + M_D \geq 300$
(5) $A_R + M_R \leq 200$
(6) $A_R + G_D = 100$

In addition, there are the constraints to indicate that the variables are non-negative:

(7) $G_R \geq 0$
(8) $A_R \geq 0$
(9) $M_R \geq 0$
(10) $G_D \geq 0$
(11) $A_D \geq 0$
(12) $M_D \geq 0$

There are six variables, so combinations of six constraints must be considered. Every equation must be included in every set of six constraints together with two of the other eight constraints. This gives ${}_8C_2 = 28$ combinations to check. The students work to eliminate some of these combinations.

Fortunately, the students are able to eliminate a number of these combinations because G_D, G_R, and A_D cannot be zero. This means there is no need to consider the equations corresponding to constraints (7), (10), and (11). Therefore the number of cases to check is ${}_5C_2 = 10$. These are listed here:

- (1), (2), (3), (6), (4), (5)
- (1), (2), (3), (6), (4), (8)
- (1), (2), (3), (6), (4), (9)
- (1), (2), (3), (6), (4), (12)
- (1), (2), (3), (6), (5), (8)
- (1), (2), (3), (6), (5), (9)
- (1), (2), (3), (6), (5), (12)
- (1), (2), (3), (6), (8), (9)
- (1), (2), (3), (6), (8), (12)
- (1), (2), (3), (6), (9), (12)

6. **Group Presentations to the Class on the *Homework: Meadows or Malls? Now We Are Ready!*** Have several groups present their ideas concerning the number of combinations that have to be considered. Why do students need to deal with just ten sets of six at the end? How did they eliminate all of the others?

7. **Small Group Work on Solving the Ten 6 X 6 Systems**: Have student groups work on solving these ten systems using the matrix capability of the graphing calculator. Once each system is solved, have students test the results in the cost equation, Cost = $50G_R + 200A_R + 100M_R + 500G_D + 200A_D + 1000M_D$.

Combination	G_R	A_R	M_R	G_D	A_D	M_D	Cost
(1), (2), (3), (6), (4), (5)	50	–150	350	250	250	–200	violates (8), (12)
(1), (2), (3), (6), (4), (8)	200	0	50	100	100	100	365,000
(1), (2), (3), (6), (4), (9)	225	25	0	75	75	150	353,750
(1), (2), (3), (6), (4), (12)	150	–50	150	150	150	0	violates (8)
(1), (2), (3), (6), (5), (8)	200	0	200	100	100	–50	violates (12)
(1), (2), (3), (6), (5), (9)	400	200	0	–100	–100	150	violates (10), (11)
(1), (2), (3), (6), (5), (12)	250	50	150	50	50	0	violates (4)
(1), (2), (3), (6), (8), (9)	200	0	0	100	100	150	410,000
(1), (2), (3), (6), (8), (12)	200	0	150	100	100	0	violates (4)
(1), (2), (3), (6), (9), (12)	[A] has no inverse.						

8. **Group Presentations to the Class on the Solution to *Meadows or Malls?*** Have several groups present their solutions to the entire class. You may need to help students answer any remaining questions. By the end of this discussion, the best solution for the use of the various parcels of land should be determined, and the problem will be solved!

Assessment

Group presentations are a very effective means of assessing student understanding. In this lesson, the video teacher uses a combination of small group work along with class discussions and group presentations to assess student understanding and to allow students to assess their own understanding. In the process, students are actually taking more responsibility for their own learning and continually assessing their own understanding.

Extensions & Adaptations

- Linear programming is a powerful modeling technique that has become an important part of applied mathematics through its usefulness in guiding quantitative decisions in business, industry, and government. As an independent project, some students could investigate the history of this relatively new mathematical topic. They could explore the work of Leonid Kanatorovich and Wassily Leontief in the areas of manufacturing schedules and economics, the applications of linear programming during World War II, and G. Dantzig's simplex method.

- At the very end of the video lesson, the students are assigned the task of writing a problem like the *Meadows or Malls?* problem. You could have your students write such problems. These problems could then be shared with the entire class or displayed on a bulletin board.

Mathematically Speaking

Appropriate use of technology is very important in mathematics classes. Technology can do wonderful things to increase our mathematical power and to bring about greater understanding. It is extremely important, however, to make certain that students understand what the calculator is doing.

Once students understand how to solve systems of equations involving two equations and two unknowns using matrices by hand, it is appropriate to introduce them to the graphing calculator in order to solve larger systems. The important steps of being able to take a system, to write it in matrix form, and then to solve the system by hand should precede work with the calculator.

What do students need to understand before they use the calculator to solve systems? They should be able to rewrite a system in matrix form. The system,

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

is the same as $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$. If students have difficulty understanding this, have them multiply the matrices on the left. First row times first column, and second row times first column will give $\begin{bmatrix} ax + by \\ dx + ey \end{bmatrix}$. Since this is equal to $\begin{bmatrix} c \\ f \end{bmatrix}$, and since corresponding elements must be equal, then we know:

$$\begin{aligned} ax + by &= c \\ dx + ey &= f. \end{aligned}$$

Students must also know that in order to solve $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$, they need to multiply both sides by the inverse of the coefficient matrix,
$\begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1}\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1}\begin{bmatrix} c \\ f \end{bmatrix}$.

They need to know how to calculate the inverse which is $\begin{bmatrix} \dfrac{e}{ae-bd} & \dfrac{-b}{ae-bd} \\ \dfrac{d}{ae-bd} & \dfrac{a}{ae-bd} \end{bmatrix}$.

From this form of the inverse, students should understand that the inverse will not be defined when $ae - bd = 0$. This will happen when the lines have the same slope. That is, the lines may have every point in common, or no points in common. In either case, the inverse will not be defined and the system will have no unique solution.

Once students understand all of the above, they are ready to take advantage of the power of the calculator, and they are also ready to interpret any error message should they receive one, such as when the system has no unique solution.

Tips From Ellen

Encouraging Participation and Building Communication Skills

It is very hard to give up center stage—most teachers fall in the category of lecture junkies. In order to promote mathematical discourse, however, it is essential to actively and explicitly promote student participation.

Following are some tips, illustrated in this lesson and others, that you can use to increase student participation and discourse, as well as opportunities for informal assessment:

Expectations:
Make whole group discussion an explicit expectation for all. Randomly select who will report by drawing the names of individuals or groups—this keeps everyone on their toes and prevents teachers from inadvertently favoring certain students or groups. Alternatively, record participation and make it part of student grades. Create ground rules for reporting, such as speaking clearly, using precise terminology, and using visuals. For group reports, involving all group members in the report is a key ground rule.

Define success in the classroom to mean that everyone understands. The job of the class is not done until every group understands. A group's work is not done until every individual understands.

Create an expectation that everyone will work together. Regroup students regularly. When grouping for a substantial period of time, consider heterogeneous groups by ability, gender, ethnicity, and learning style. One school of thought says that you should try not to isolate one boy in a group of girls, one African-American student in a group of Caucasian students, and so on. Since girls of high school age may defer to boys, some teachers will frequently assign same-sex groups to provide opportunities for girls to assume group leadership positions.

Risk Free Environment:
Provide rehearsal time in pairs or small groups prior to large group reporting. This may involve reviewing homework in small groups, assigning group problems, or using a Think-Pair-Share structure.

Ask students to recap directions before beginning an activity. This increases student attention to directions and allows for corrections so that everyone is on the right track.

Ask students to explain previously learned vocabulary and concepts. This increases student attention, allows for assessment of learning, and permits students to shine.

Dignify answers. Ask open ended questions and emphasize effective problem solving strategies rather than correct answers. Ask if anyone else would like to add anything or has a different idea. If appropriate, allow students to summarize their current thinking based on follow-up discussion.

Use choral response as appropriate. It is validating to everyone, builds community, and encourages some students to participate.

Questioning Techniques:
Extend student thinking by paraphrasing responses and inviting additional comments. Use questions like, "So what I'm hearing from your group is ... is there any group that has a similar result? Does any group disagree? How might you explain that? Can someone summarize our current thinking on that point?"

Even if you think students have "gotten it," continually ask them to paraphrase, to summarize, to explain it in a different way or form. This allows you to check for understanding, attend to different learning styles, and reinforce learnings.

Resources

Introduction and Implementation Strategies for the Interactive Mathematics Program, A Guide for Teacher-Leaders and Administrators. Berkeley, CA. Key Curriculum Press, 1997.

Key Curriculum Press, P.O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH. E-mail address: sales@keypress.com, http;//www.keypress.com

Internet location: **http://www.c3.lanl.gov/mega-math/workbk/logic/loplay.html**
Logic and reasoning are an important part of the video lesson and an important part of the study of mathematics. In the lesson, students use reasoning skills to eliminate some of the constraints that must be considered in order to solve the problem. At this location, students also must use reasoning skills to figure out the answer to a mystery. The problem is presented in the form of a play.

Internet location: **http://complex.csu.edu.au/complex/**
Complex Systems is a new branch of mathematics that includes disciplines like chaos theory, nonlinear systems, and neural networks. This site has some interesting hypermedia tutorials on subjects like L-Systems (rules and symbols used to model growth processes), fuzzy logic, fractals, and ideas like cellular automata. The location also includes a gallery of fractal images (like ferns) and a cellular automata demonstration graphic called Mona Lisa Burning.

Internet location: **http://www.cut-the-knot.com/content.html**
Students can access a wide variety of puzzles and mathematical facts at this *Interactive Mathematics Miscellany and Puzzles* site. This site has won numerous awards and has many interesting things to explore. Please note that this site is not related to the Interactive Mathematics Program.

Internet Location: **http://www.mcs.anl.gov/home/otc/Guide/faq/linear-programming-faq.html.** This is a rich source of information about linear programming. It includes a list of frequently asked questions along with answers, such as "What is Linear Programming?", "Where is there good software to solve LP problems?", "What is a modeling language?", "What software is there for Network models?", "What software is there for the Traveling Salesman Problem (TSP)?", and "What software is there for the Knapsack Problem?"

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

1. The video lesson is part of a series of lessons that make up the *Meadows or Malls?* unit. Based on what you see the students doing in this video, name some of the benefits of this type of focus over a longer period of time. How can you use this idea to strengthen your lessons?

Standard 2: The Teacher's Role in Discourse

2. The video teacher seems to take a back seat in this lesson. Many students and parents might accuse her of not doing her job and not teaching because she is not up in front of the class telling the students exactly how to solve the problems. How do you view her role? How would you persuade students and parents to view such a role in a positive way?

Standard 3: Students' Role in Discourse

3. The young women outnumbered the young men in this class. How did this affect the interaction in the classroom?

Standard 4: Tools for Enhancing Discourse

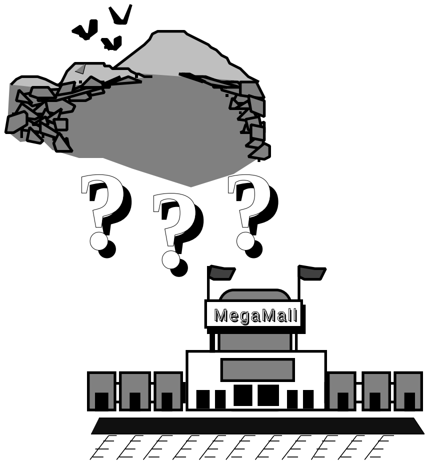
4. At certain times students were given overhead transparencies and markers in order to prepare for making a presentation. How well did this seem to work? What tools do you use to help save time and facilitate group presentations in your classroom?

5. Why were the graphing calculators an important part of this lesson? Could the lesson have been taught without them? Would that have been better? Why or why not?

Standard 6: Analysis of Teaching and Learning

6. In the video lesson, students gave group presentations and were asked to write a similar type of linear programming problem along with the solution. Is this enough assessment? What other assessment would you expect to be used?

Meadows or Malls?



Who would have thought that so much good fortune could cause so much trouble? Well, this time it sure did in River City. Actually, there were three separate pieces of good fortune.

First, Mr. Goodfellow died and left his 300 acre farm to the city. His will had no stipulations, so the city could do what ever they wished with it.

Then the U.S. Army closed its base on the edge of town. It was only 100 acres, but it had a lovely view of the both the river and the city. Again, this was given to the people of River City by the U.S. government to use in any way they chose.

Finally, there was 150 acres of land that had been leased to a mining company 99 years ago. Well, the lease was up, and since the company had never found enough minerals there to make a profit, they did not wish to renew their lease. So that land was given back to the city with no restrictions on its use.

That was 550 acres of land that the city could use in any way it decided. The problem was that a city isn't exactly an "it." A city contains many people who don't always agree. The people in River City did not agree on how to use the 550 acres.

Essentially, there were two sides to the controversy. One side wanted to use as much of the land as possible for *development*; that is, for stores, businesses, and housing. The other side of the controversy wanted to use as much of the land as possible for *recreation*. That is, they wanted park land, hiking trails, a wildlife preserve, and picnic areas.

The Chamber of Commerce won an initial victory by getting the city council to agree that at least 300 acres would go for development. The Chamber of Commerce also thought that the more attractive property of the army base and mining land should go to development, while any recreation land could come from Mr. Goodfellow's property. But the Sierra Club felt that some of the more attractive land should go for recreation. The two groups finally came up with a two-part compromise:

- At most 200 acres of the army base and mining land could go for recreation.

- The amount of army base land used for recreation plus the amount of land from Mr. Goodfellow used for development had to add up to exactly 100 acres.

Everyone realized that the city would have to improve any land used for development, putting in sewers, streets, electricity, etc. The city would also have to put some money into any land used for recreation. The city manager made the chart below to show, for each parcel, how much each type of land use would cost the city.

Parcel	Improvement costs per acre for recreation land	Improvement costs per acre for development land
Goodfellow's	$50	$500
Army Base	$200	$2,000
Mining Land	$100	$1,000

Everyone agreed that they wanted to keep the cost to River City at a minimum, while satisfying their needs.

So the matter was turned over to the city manager. She was directed to decide how to split the land use between development and recreation in a way that would minimize the cost to the city of the necessary improvements, at the same time making sure that at least 300 acres went for development and that the two-part compromise was followed.

Now, she had tackled some complicated problems in her time, but this seemed a bit much for her to handle. So she turned the matter over to a consulting firm of city planners. Your group is to function as that consulting firm. You will be working on this problem over the course of this unit.

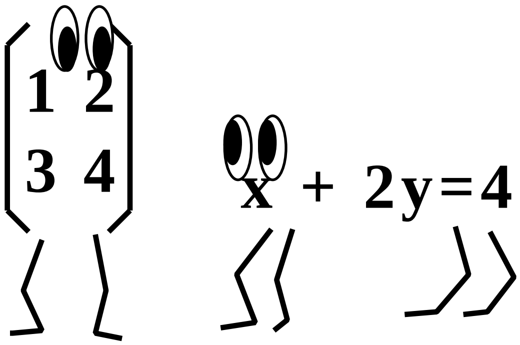
Today, you want to find out as much as you can about the problem. By the end of the unit, you will find the best solution.

1. Find one way to allocate the land and satisfy the constraints. Find the cost to the city for this solution even though you may know that it is not the least costly allocation.

2. Note any facts that you discover about the problem, such as "There must be at least ____ acres of Mr. Goodfellow's land used for _____." Explain why your statements are facts.

3. What approaches to solving this problem would you like to try if you had more time? What approaches did you try that didn't seem to work?

Inverses and Equations



1. Try to solve each of these systems of linear equations:

   a. $$\begin{aligned} x + 2y &= 4 \\ 3x + 4y &= 8 \end{aligned}$$

   b. $$\begin{aligned} x + 2y &= 2 \\ 3x + 4y &= 1 \end{aligned}$$

   c. $$\begin{aligned} x + 2y &= 4 \\ 2x + 4y &= 8 \end{aligned}$$

   d. $$\begin{aligned} x + 2y &= 2 \\ 2x + 4y &= 1 \end{aligned}$$

2. Now try to find the inverse of each of these matrices: (These are the coefficient matrices for the systems of equations in Question 1.)

   a. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

   b. $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

3. Did you get inverses in both cases in Question 2? How are your results in Question 2 related to those in Question 1?

TI's Again

You already saw that the TI calculator can be used to multiply matrices.

The TI has another wonderful talent: it can calculate multiplicative inverses of matrices (when they exist).

And it's easy to get the TI to do this, once you have the matrix in the TI's memory.

For example, suppose you have given the TI the entries for some matrix [A].

To get the inverse, press 2nd [A] 1 , the x^{-1} key, and ENTER . The inverse of [A] will appear on the screen.

Of course, that will only work if the matrix has an inverse. If not, the screen will say "**ERROR 05 MATH**." If you then press 1 (for "**Go to Error**"), the TI will highlight the "$^{-1}$" on the main screen, telling you where the problem is. In this case, you can't use a matrix inverse to solve the system of linear equations. That's fine, because when the coefficient matrix has no inverse, the system of equations doesn't have a unique solution.

But, usually, that won't happen and the inverse will just appear in front of you. So if you have a system of linear equations that is represented by a matrix equation like [A] [X] = [B], you know that the solution matrix is just $[X] = [A]^{-1}\,[B]$. So all you have to do to solve the system is enter the matrices, and push a few keys.

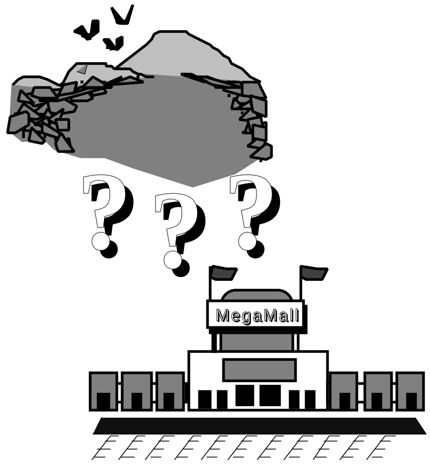
So try to solve these linear systems without doing any arithmetic, letting the TI do all the hard work for you. You should check your solutions to at least the first system, to make sure you are doing the process correctly.

1. $$\begin{aligned} 5d + 2e &= 11 \\ d + e &= 4 \end{aligned}$$

2. $$\begin{aligned} 2r + 3s - t &= 3 \\ r - 2s + 4t &= 2 \\ 4r - s + 7t &= 8 \end{aligned}$$

3. $$\begin{aligned} 4w + x + 2y - 3z &= -16 \\ -3w + x - y + 4z &= 20 \\ -w + 2x + 5y + z &= -4 \\ 5w + 4x + 3y - z &= -10 \end{aligned}$$

Meadows or Malls?: Now We Are Ready!



About a year or two ago (it seems), you started working on a problem called *Meadows or Malls?*

You now have all the tools to solve that problem, although it still will take some work to answer the main question in that problem.

But it has been a while since you thought about linear programming instead of matrices. So, in preparation for resuming work on that problem, do the following:

1. Write all the constraints involved in the *Meadows or Malls?* problem.

2. Review your general strategy for solving linear programming problems, and discuss how this will apply to *Meadows or Malls?*

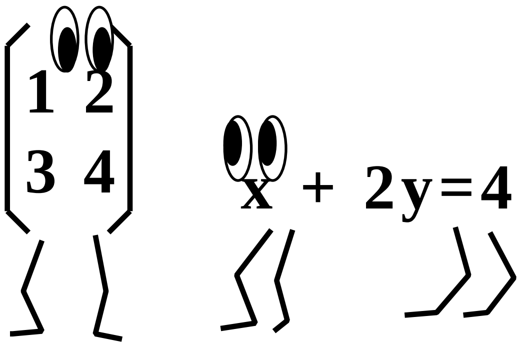
The *Meadows or Malls?* problem has twelve constraints, of which four are equations and eight are inequalities.

That gives a lot of combinations to check to find corner points for the feasible region.

3. Make a list of all of the combinations that you need to check. Remember that the equations must be part of each combination. You should refer to the constraints by number in listing your combinations.

4. Think about ways to reduce this list. For instance, it is possible to show that certain of the variables can't be zero. See if you can prove this.

Inverses and Equations *(Selected Answers)*



1. Try to solve each of these systems of linear equations:

   a. (0, 2)

   b. $(-3, \frac{5}{2})$

   c. *These two equations represent the same line. They have every point in common.*

   d. *These two lines are parallel. They have no point in common.*

2. Now try to find the inverse of each of these matrices: (These are the coefficient matrices for the systems of equations in Question 1.)

   a. $\begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$

   b. *There is no inverse. The calculator gives an error message.*

3. *No, part (b) does not have an inverse. There is an inverse only when there is a unique solution.*

TI's Again

1. (1, 3)

2. *No solution.*

3. (22, –29, 9, 31)

Meadows or Malls?: Now We Are Ready!

Selected Answers

1. The first six constraints come from the problem situation:

   (1) $G_R + G_D = 300$
   (2) $A_R + A_D = 100$
   (3) $M_R + M_D = 150$
   (4) $G_D + A_D + M_D \geq 300$
   (5) $A_R + M_R \leq 200$
   (6) $A_R + G_D = 100$.

   In addition, there are the constraints to indicate that the variables are non-negative:

   (7) $G_R \geq 0$
   (8) $A_R \geq 0$
   (9) $M_R \geq 0$
   (10) $G_D \geq 0$
   (11) $A_D \geq 0$
   (12) $M_D \geq 0$.

2. The strategy should be something like:

   List the constraints, including constraints that make each variable non-negative. These are listed in the previous problem.

   Consider all combinations of constraints, taken *n* at a time. For each combination, look for a common solution (an ordered *n*-tuple) to the corresponding equations. For this problem, that means six constraints taken at a time. Since constraints (1), (2), (3), and (6) are equations, they must be included in every set of six, together with two of the other eight constraints.

   If the combination has a common solution, see whether the solution fits *all* of the original constraints.

   If a common solution fits all the constraints, evaluate the *profit* or *cost* function (i.e., the function being maximized or minimized) at that common solution. For this problem, the cost is to be minimized.

   $$C = 50G_R + 200A_R + 100M_R + 500G_D + 2000A_D + 1000M_D$$

   Among the common solutions tested that have been evaluated, identify the one that maximizes or minimizes the given function.

3-4. Fortunately, you can eliminate a number of these combinations because G_D, G_R, and A_D cannot be zero. This means these is no need to consider the equations corresponding to constraints (7), (10), and (11). Therefore the number of cases to check is ${}_5C_2 = 10$.

Combination	G_R	A_R	M_R	G_D	A_D	M_D	**Cost**
(1), (2), (3), (6), (4), (5)	50	–150	350	250	250	–200	violates (8), (12)
(1), (2), (3), (6), (4), (8)	200	0	50	100	100	100	365,000
(1), (2), (3), (6), (4), (9)	225	25	0	75	75	150	353,750
(1), (2), (3), (6), (4), (12)	150	–50	150	150	150	0	violates (8)
(1), (2), (3), (6), (5), (8)	200	0	200	100	100	–50	violates (12)
(1), (2), (3), (6), (5), (9)	400	200	0	–100	–100	150	violates (10), (11)
(1), (2), (3), (6), (5), (12)	250	50	150	50	50	0	violates (4)
(1), (2), (3), (6), (8), (9)	200	0	0	100	100	150	410,000
(1), (2), (3), (6), (8), (12)	200	0	150	100	100	0	violates (4)
(1), (2), (3), (6), (9), (12)	[A] has no inverse.						

The least expensive solution for the city is:

$G_R = 225$

$A_R = 25$

$M_R = 0$

$G_D = 75$

$A_D = 75$

$M_D = 150.$

Since cost is given by $C = 50G_R + 200A_R + 100M_R + 500G_D + 2000A_D + 1000M_D$, the least expensive cost to the city is \$353,750.